Pr. Find the phase velocity and the magnitude of the attenuation constant of plane waves at a frequency 10 GHz in polyethelene, given

$$\mu = \mu_0, \quad \varepsilon_r = 2.3 \text{ and } \sigma = 2.56 \times 10^{-4} \frac{\text{mho}}{\text{m}}$$

Phase velocity

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{\sqrt{\varepsilon_0/\varepsilon}}{\sqrt{\mu_0\varepsilon_0}} = c\sqrt{\varepsilon_0/\varepsilon}$$

$$= c\sqrt{\frac{1}{\varepsilon_r}} = 3\times10^8 \times \sqrt{\frac{1}{2.3}}$$

$$v = 1.97\times10^8 \text{ m/s}$$

Since $$\frac{\sigma}{\omega\varepsilon} = \frac{2.56\times10^{-4}}{2\pi\times10^{10}\times2.3\times8.85\times10^{-12}} = 2\times10^{-4}$$

is much smaller than 1

So $$\beta = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\frac{\sigma^2}{2\varepsilon^2\omega^2}\right]^{1/2}$$

$$= \frac{1}{2}\sigma\sqrt{\frac{\mu}{\varepsilon}} = \frac{1}{2}\frac{\sigma}{\omega\varepsilon}\cdot\omega\sqrt{\mu\varepsilon}$$

$$= \frac{1}{2}\frac{\sigma}{\omega\varepsilon}\cdot\frac{\omega}{v}$$

$$= \frac{1}{2}\times2\times10^{-4}\times2\pi\times10^{10}\times\frac{1}{1.97\times10^8}$$

$$\beta = 3.18\times10^{-2}$$

Reflection from the surface of a conductor (metal)

We extend the previous arguments to a boundary surface of a conducting medium. We consider the simple case of normal incidence.

For incident, reflected and transmitted waves,

$$E_I = E_{0I} \exp\{i(k_I \cdot r - \omega t)\}; \quad H_I = \frac{k_I \times E_I}{\omega \mu_1} \quad -(1)$$

$$E_R = E_{0R} \exp\{i(k_I \cdot r - \omega t)\}; \quad H_R = \frac{k_I \times E_R}{\omega \mu_1} \quad -(2)$$

$$E_T = E_{0T} \exp\{i(k_T \cdot r - \omega t)\}; \quad H_T = \frac{k_T \times E_T}{\omega \mu_2} \quad -(3)$$

Since the medium 2 is a conducting medium the propagation vector k_T is given by

$$k_T^2 = \varepsilon_2 \mu_2 \omega^2 \left[1 + \frac{i\sigma}{\varepsilon_2 \omega}\right]$$

The boundary conditions require that

$$E_{0I} - E_{0R} = E_{0T} \quad -(4)$$

$$k_I (E_{0I} + E_{0R}) = k_T E_{0T} \quad -(5)$$

Since k_T is complex, E_{0R} and E_{0T} cannot both be real hence, one should expect phase shifts other than 0 or π ~~is~~ in the reflected and transmitted waves.

Solving eq (4) and (5), we have

$$E_{0R} = \frac{k_T - k_I}{k_T + k_I} E_{0I} \quad -(6)$$

$$E_{0T} = \frac{2k_I}{k_T + k_I} E_{0I}$$

Substituting $k_I = \omega(\varepsilon_1 \mu_1)^{1/2}$ and k_T

$$E_{OR} = \frac{\sqrt{\varepsilon_2 \mu_2 \omega^2}\left(1+\frac{i\sigma}{\varepsilon_2 \omega}\right)^{1/2} - \omega(\varepsilon_1 \mu_1)^{1/2}}{\sqrt{\varepsilon_2 \mu_2 \omega^2}\left(1+\frac{i\sigma}{\varepsilon_2 \omega}\right)^{1/2} + \omega(\varepsilon_1 \mu_1)^{1/2}} E_{OI} \quad (7)$$

and

$$E_{OT} = \frac{2\omega(\varepsilon_1 \mu_1)^{1/2}}{\sqrt{\varepsilon_2 \mu_2 \omega^2}\left(1+\frac{i\sigma}{\varepsilon_2 \omega}\right)^{1/2} + \omega(\varepsilon_1 \mu_1)^{1/2}} E_{OI} \quad (8)$$

Let us now examine the case of a perfect conductor for which $\sigma = \infty$, Then

$$E_{OR} = E_{OI} \text{ and } E_{OT} = 0$$

Hence, the ~~reflectance~~ reflection is complete.
If the conductor is not a perfect conductor but a very good conductor $\frac{\sigma}{\varepsilon_2 \omega} >> 1$.
The approximation gives

$$k_T = \alpha + i\beta$$
$$= (1+i)\sqrt{\frac{\omega\sigma\mu_2}{2}} = \frac{1+i}{\delta}$$

we have made use of the definition of skin depth δ.
Substituting this in eq^n (6)

$$E_{OR} = \frac{\frac{1+i}{\delta} - \omega(\varepsilon_1 \mu_1)^{1/2}}{\frac{1+i}{\delta} + \omega(\varepsilon_1 \mu_1)^{1/2}} E_{OI}$$

$$E_{OR} = \frac{\left\{\frac{1}{\delta} - \omega(\varepsilon_1 \mu_1)^{1/2}\right\} + \frac{i}{\delta}}{\left\{\frac{1}{\delta} + \omega(\varepsilon_1 \mu_1)^{1/2}\right\} + \frac{i}{\delta}} E_{OI}$$

Therefore, the reflection coefficient R is given by

$$R = \frac{|E_{0R}|^2}{|E_{0I}|^2} = \frac{\{1 - \omega(\varepsilon_1\mu_1)^{1/2}\delta\}^2 + 1}{\{1 + \omega(\varepsilon_1\mu_1)^{1/2}\delta\}^2 + 1}$$

Because $\frac{\sigma}{\varepsilon_2\omega} \gg 1$, $\omega(\varepsilon_1\mu_1)^{1/2}\delta \ll 1$, hence

$$R \simeq 1 - 2\omega(\varepsilon_1\mu_1)^{1/2}\delta \quad \text{---(9)}$$

$$= 1 - 2\sqrt{\frac{2\omega\varepsilon_1\mu_1}{\sigma\mu_2}} \quad \text{---(10)}$$

If the magnetic permeabilities are assumed to be equal

$$R = 1 - 2\sqrt{\frac{2\omega\varepsilon_1}{\sigma}} \quad \text{---(11)}$$

The measure of energy transmitted into the conducting medium is obtained by calculating the transmission coefficient T

$$T = 1 - R = 2\sqrt{\frac{2\omega\varepsilon_1}{\sigma}} \quad \text{-(12)}$$

Magnitude of the fraction of energy transmitted in a good conductor, we calculate T for copper

$\sigma = 6\times10^{7}\,(\text{ohm m})^{-1}$, $\nu = 10^{10}\,\text{sec}^{-1}$

$$T = 2\sqrt{\frac{2\times2\pi\times10^{10}\times8.85\times10^{-12}}{6\times10^{7}}}$$

$$\simeq 3\times10^{-4}$$

Which is extremely small for direct measurement to be made